A robust affine invariant similarity measure based on visibility

Michiel Hagedoorn* Mark Overmars Remco C. Veltkamp
Department of Computer Science, Utrecht University
Padualaan 14, 3584 CH Utrecht, The Netherlands
{ mh, markov, Remco.Veltkamp }@cs.uu.nl

Abstract

Measures of similarity are fundamental in the field of geometric pattern recognition. It is desirable that such measures are metrics. Little is known about metrics on 1-dimensional disconnected planar sets. Until recently, the only reasonable metric for such sets seemed to be the Hausdorff metric. However, the usefulness of this metric is limited by the fact that it is not "robust for outliers". In addition, the Hausdorff distance is only invariant under isometric transformations.

Using the concept of visibility, we define the *reflection metric*. This affine invariant metric is robust for various types of pattern distortions including outlying points. The computation of the reflection metric involves the construction of *reflection-visibility partitions*. Using these structures, the reflection metric can be computed in $O(r(n_A + n_B))$ randomised time, for two unions of line segments, with n_A and n_B line segments, separately, where r is the complexity of the overlay of two partitions.

1 Introduction

The visibility from a particular viewpoint in a pattern gives a local description of the pattern. The visibilities from all possible viewpoints give a complete representation of the pattern. Visibility is defined in terms of affine geometry, the concept does not depend on Euclidean distances. Therefore, visibility can be used as a tool for affine invariant shape recognition and affine pattern matching. We use a strong type of visibility to compute an affine invariant pattern metric, called the reflection metric.

Our approach uses a special form of visibility, called reflection-visibility, for defining a similarity measure on line patterns. This measure, the reflection metric, is affine invariant by definition. That is, the distance between affine transformed patterns $t(A)$ and $t(B)$ equals the distance between the original A

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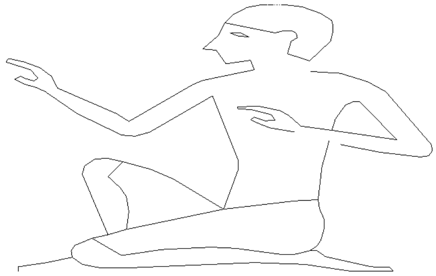

Figure 1: A pattern $A \in \mathcal{R}^2$.

Figure 2: The function ρ_A.

and B. The reflection metric is robust. It responds proportionally when lines are deformed a little, slightly translated copies of existing lines are added, small cracks are made in lines, and small new lines are added.

To compute the reflection metric we need to know the structure of the visibility partition corresponding to a special type of visibility. Visibility partitions consists of equivalence classes with constant combinatorial visibility. Plantinga and Dyer [11] call this structure the *viewpoint space partition*. The dual of the visibility partition is called the *aspect graph*, see Kriegman and Ponce [10], Bowyer and Dyer [4], and Gigus et al. [6]. The number of possible views, the size of the visibility partition, was investigated, under varying assumptions, by Agarwal and Sharir [1], and de Berg et al. [5]. For polygons, results about visibility partitions were found by Guibas et al. [8], Aronov et al. [2], and Bose et al. [3].

2 The reflection metric

The reflection metric, introduced in [9], defines a distance between finite unions of algebraic curve segments in the plane. If A and B are such unions, the reflection distance is denoted as $d_{\mathtt{R}}(A, B)$. The reflection metric turns the patterns A and B into functions $\rho_A, \rho_B : \mathbb{R}^2 \to \mathbb{R}$, after which the integrated absolute difference of these functions is normalised. The functions are defined using a strong form of visibility, called reflection-visibility.

Figure 1 shows a two-dimensional pattern A consisting of a finite number of line segments. Figure 2 shows the corresponding function $\rho_A : \mathbb{R}^2 \to \mathbb{R}$ represented as a grey-scale image, in which black corresponds with value 0. The examle pattern is hieroglyphic "A1" obtained from the hieroglyphica sign list, see [7].

Let T be the group of affine transformations on $\mathbb{R}^2$. The reflection metric is invariant for T, meaning that $d_{\mathtt{R}}(t(A), t(B)) = d_{\mathtt{R}}(A, B)$ for any affine transformation $t \in T$. As a result, $d_{\mathtt{R}}$ can be used to construct a metric on *affine*

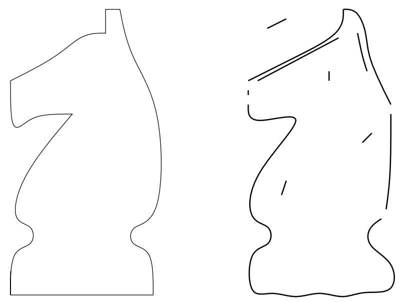

Figure 3: Deformation, blur, cracks and noise.

shapes, patterns modulo affine transformation:

$$D_R(T(A), T(B)) = \inf_{t \in T} d_{\mathtt{R}}(t(A), B).$$

The reflection metric is robust for various types of effects caused by discretisation and unreliable feature extraction. Slight *deformations* of patterns only increase or decrease distances slightly. Introducing *blur*, by adding new lines near existing lines in a pattern, only causes a proportional change in the reflection distance. Making *cracks* in the interior of lines, splitting them up into multiple smaller ones, only changes the distance proportional in the length of these cracks. Adding *noise* in the form of new lines, far away from other lines, only increases the distance proportionally to the length of the added line. Figure 3 illustrates the effects of deformation, blur, cracks, and noise. The left pattern is the "original", the right pattern is "affected" by the four types of distortion.

The reflection metric can be computed by constructing and traversing an arrangement, which is the overlay of two so-called reflection-visibility partitions. Figure 4 is a schematic drawing of a reflection-visibility partition.

Theorem 1 *Let A and B each be unions of n_A and n_B segments, respectively. Using randomisation, the reflection distance $d_{\mathtt{R}}(A, B)$ can be computed in $O(r(n_A + n_B))$ time, where r is the complexity of the overlay of the reflection-visibility partitions of A and B.*

3 Conclusion

We presented a new metric for pattern matching, the reflection metric. This metric is invariant under the group affine transformations and can therefore be used for affine shape recognition. The reflection metric is robust for pattern-defects such as deformation, blur, cracks, and noise. It can be generalised to finite unions of algebraic hyper-surface patches in any dimension.

The reflection metric is defined in terms of reflection visibility. Trans-visibility and reflection-visibility are stronger than visibility. Reflection-visible

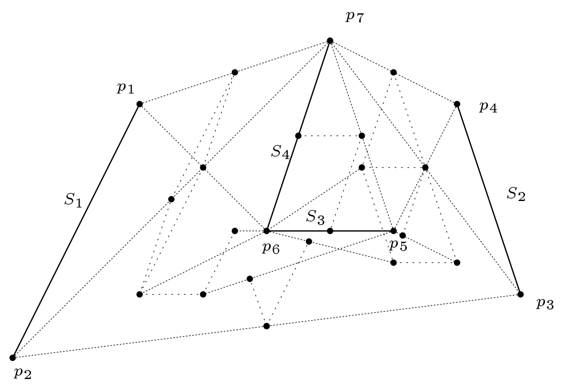

Figure 4: A reflection-visibility partition.

points are always trans-visible, and trans-visible points are always visible. We analysed the partitions corresponding to the visibilities, starting at normal visibility, proceeding with trans-visibility, and ending with reflection-visibility. In both steps new types of events emerged, making the resulting partitions more complex.

Constructions show that the worst-case combinatorial complexity for each of the three types of partitions is $\Omega(n^4)$. Using randomised incremental construction, each of the three corresponding arrangements can be built in $O((n + k)\log(n) + v)$ time, where k is the number of visibility edges, and v is the number of intersections in the arrangement. The structure of reflection-visibility partitions can be used to compute the reflection metric for two collections of segments in $O(r(n_A + n_B))$ randomised time, where r is the complexity of the overlay of two reflection-visibility partitions.

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