Approximate Center Points in Dense Point Sets

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1 Center Points

The notion of center points is one way to generalize the median of finite ordered point sets to higher dimensions. Let $P = \{p_1, \ldots, p_n\} \subset \mathbb{R}^d$ be a set of points in the d-dimensional Euclidean space. A point $c \in \mathbb{R}^d$ is a β-*center* ($\beta \in]0, \frac{1}{2}]$) of P, if every hyperplane h containing c is β-*splitting* P, that is, there are at least $\lceil n\beta \rceil$ points of P in each closed halfspace determined by h. Since both of these halfspaces contain the points $P \cap h$, we do not have to pose a general position assumption to handle extreme configurations[1]. The set of $1/(d+1)$-centers is simply called the *center* of P.

It is shown in [2] that a center always exists. But unlike the one-dimensional median, in general one can not expect to find points of P in the center. Imagine, e.g., all points of P distributed on the perimeter of a circle, then no point of P belongs to the center. An example of the center of 5 points is depicted in Figure 1.

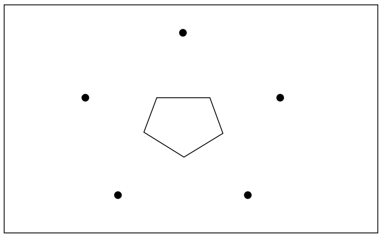

Abbildung 1: Center of 5 points distributed near a circle.

For the planar case ($d = 2$) a point in the center can be computed in linear time [4]. Using a randomized approach [1], an approximate center can be computed via iterated Radon points. The algorithm computes a β-center with $\beta \in \Omega(1/d^2)$ and error probability at most $1/n$ in time subexponential in d.

[1]For example, if all points of $P \subset \mathbb{R}^2$ lie on the same line l, then l is 1-splitting P.

2 Dense Point Sets

To define *density*, the structure of a distance function is necessary. In this paper we always use $\mathbb{R}^d$ equipped with the Euclidean metric $\|\cdot\|$ and denote this Euclidean space by $\mathbb{E}^d$.

A set of n points P in $\mathbb{E}^d$ is called δ-*dense* ($\delta > 0$), if the ratio of the maximum to the minimum distance between two points in P is at most $\delta n^{1/d}$. Arbitrarily large dense sets can be constructed as the intersection of a grid with a disk. Thus, it is equivalent to say that the minimum distance is a given constant and the maximum distance is bounded linearly by the diameter of a set of n points packed as closely as possible (respecting the minimum distance). In $\mathbb{E}^2$, $\delta \approx 1.05$ is sufficient to admit arbitrarily large δ-dense sets.

Dense point sets are described in [3]. They are relevant in practice, since in many interesting applications, e.g. motion planning in factories, density of the environment can be assumed. Contrary to this, in theoretical worst-case analysis often large distance ratios are constructed [5, page 107], sometimes exponential in the number of points, to obtain the extremal complexity of an algorithm.

3 Solution

We describe a simple, yet efficient algorithm to compute an approximate center point for a δ-dense set P in time $O(dn)$. The quality of the approximation depends on δ and exponentially on the dimension d, where the absolute value of the exponent is in $\Omega(-d \log d)$. We also present an iteration method with a linear rate of convergence, which computes an improved $1/\big(4(d-1)(\delta+1)^d\big)$-center point for sufficiently large n in worst-case time $O(n \log n)$.

Literatur

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