# Visibility Edges, Mixed Edges, and Unique Triangulation

Jay Bagga [1], Laxmi Gewali [2], and Simeon Ntafos [3]

**Abstract:** Let $P$ be a simple polygon with $n$ vertices $v_1, v_2, v_3, ..., v_n$. The edges formed by connecting the vertices of $P$ can be distinguished into four categories: (i) **polygonal edges**, (ii) **internal visibility edges**, (iii) **external visibility edges**, and (iv) **mixed edges**. Edges connecting consecutive vertices are polygonal edges; and those connecting non-consecutive vertices and lying completely inside (respectively, outside) $P$ are internal (respectively, external) visibility edges. The edges that intersect with polygonal edges are mixed edges. We show that the bounds on the number of internal, external, and mixed visibility edges is closely related to the visibility properties of uniquely triangulated polygons and report the following results. (a) the total number of internal visibility edges $i$ is always in the range $n-3 \leq i \leq \binom{n}{2} - n$, (b) the total number of external visibility edges $e$ is always greater or equal to $n-h$, where $h$ is the number of vertices lying on the boundary of the convex hull of the polygon, and (c) the number of convex vertices $c$ of a uniquely triangulated polygon lies in the range $6-n \leq 2c-n \leq 2$ if $n$ is even, and in the range $6-n \leq 2c-n \leq 3$, if $n$ is odd. We have constructed example polygons to show that the above stated bounds are sharp. Finally, we consider the bounds for $i+e$ and conjecture that it must be at least $(n-3) + \lfloor \frac{n-2}{2} \rfloor$.

---

[1]Department of Computer Science, Ball State University, Indiana, email: jay@bsu-cs.bsu.edu

[2]Department of Computer Science, University of Nevada, Las Vegas, e-mail: laxmi@cs.unlv.edu

[3]Computer Science Program, University of Texas at Dallas, e-mail: ntafos@utdallas.edu