Surfaces based on Voronoi Diagrams

Gerald Farin
Department of Computer Science
Arizone State University
Tempe, USA
email: farin@asu.edu

We introduce an interpolant due to R. Sibson that is built directly on the concept of Voronoi diagrams or Drichlet tesselations, without making reference to a Delaunay triangulation.

Sibson's interpolant is a simple scattered data interpolant with the following properties:

1. It is the natural generalizations of univariate piecewise linear interpolation.
2. It is C^1 everywhere except for the data sites where it is only C^0.
3. It has local support.
4. It has linear precision.

We present a simplification of Sibson's original formula, which should allow faster computation.

We have also investigated the following: instead of considering a discrete set of data sites, check what happens if all data sites are along a given closed boundary curve, and also let the number of data sites go to infinity. The interpolant now becomes transfinite – finite sums in Sibson's interpolant become convolution integrals for the transfinite case. The limits for these integrals have to be computed using the continous Voronoi diagram of the boundary, a structure closely related to its medial axis.

For the special case of a circular boundary, we obtain parametric surfaces that are close to minimal. If we replace the planar circular domain by a spherical cap, we are able to reproduce Enneper's minimal surface exactly, but we only have a numerical "proof" for this.