Combinatorial Structure of Dirichlet Partitions

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1 Abstract

Any finite set S of n points on the plane gives rise to a cellular decomposition of the plane called the *Voronoi partition*. It is one of the most useful construction associated with such a configuration of points and several algorithms that compute this geometrical structure are well-known. [4]

Infinite sets of points, if they are discrete, also define a partition of the plane ([1]), but can not, in general, be computed, as the known algorithms deal only with finite sets of points.

In the particular case that this infinite set is of the form GS, with G a crystallographic group and S a finite set of points then, due to the periodic structure of the set, the Voronoi partition has also a periodic structure and can be computed by computing only the Voronoi partition of a certain finite subset of GS [3].

We have implemented the algorithm ([3]) which computes periodic Voronoi partition in the plane.

When the set S reduces to one point, the resulting Voronoi partition $Vor(GP)$ is a *Dirichlet partition* . Each Voronoi region D_{gP} corresponding to $gP, g \in G$ is called a *Dirichlet domain* with center gP .

If P has trivial stabilizer ($gP \neq P, \forall g \in G \setminus \{e\}$) then it happens that the corresponding Dirichlet domain D_P is a *fundamental domain* for the crystallographic group G ([5]).

Usually, a crystallographic group is given by means of a set of generators. Moreover, a certain fundamental domain is also fixed "a priori".

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In this work in progress we study all possible combinatorial structures of Dirichlet domains according to the position of the point P inside a given fundamental domain. In this way we also find all the combinatorial types of Dirichlet domains for crystallographics groups in the plane which are fundamental domains.

For instance, let us consider the crystallographic group $p4$ having the fundamental domain the quadrilateral ABCD. We have 3 types of Dirichlet domains for this group, according to the position of the generator point P inside ABCD.

1. Fig. 1. - P is on the border of ABCD then the Dirichlet domain is a triangle .
2. Fig. 2. - P is on the diagonal BD then Dirichlet domain is quadrilateral. (ABCD)
3. Fig. 3. - neither of the above cases then the Dirichlet domain is pentagon.

It is only known that the number of edges of a Dirichlet domain is at most 6 ([5]). We intend to study the variation in number of edges when moving the generator point P.

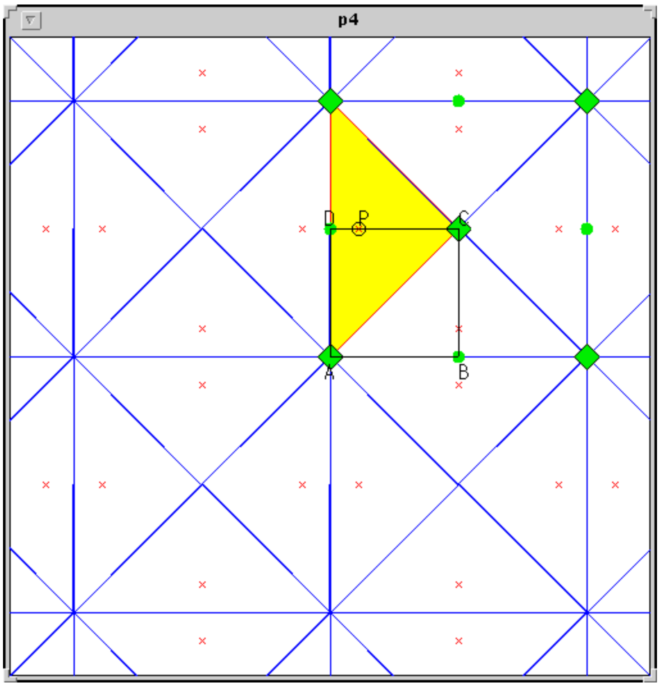

Fig. 1 P is on the border of ABCD, then the shaded triangle is the Dirichlet domain.

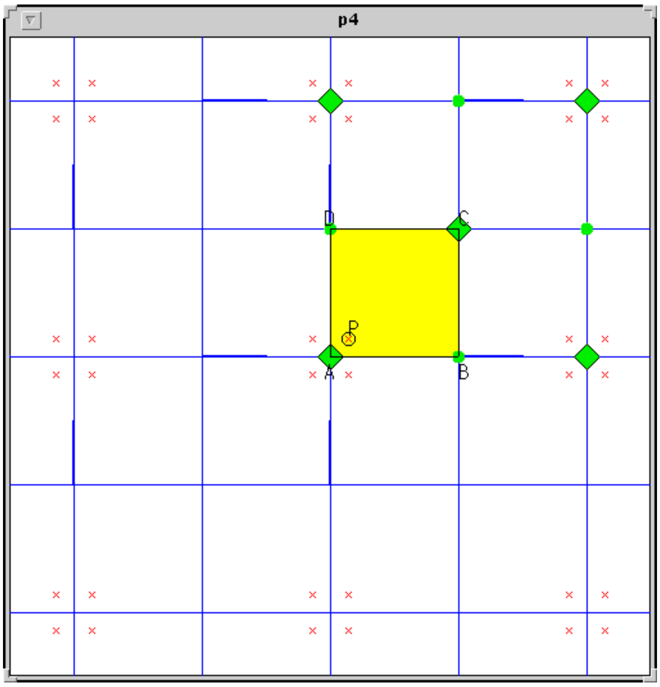

Fig. 2 P is on the diagonal which pass through the 4-center, then ABCD is the Dirichlet domain.

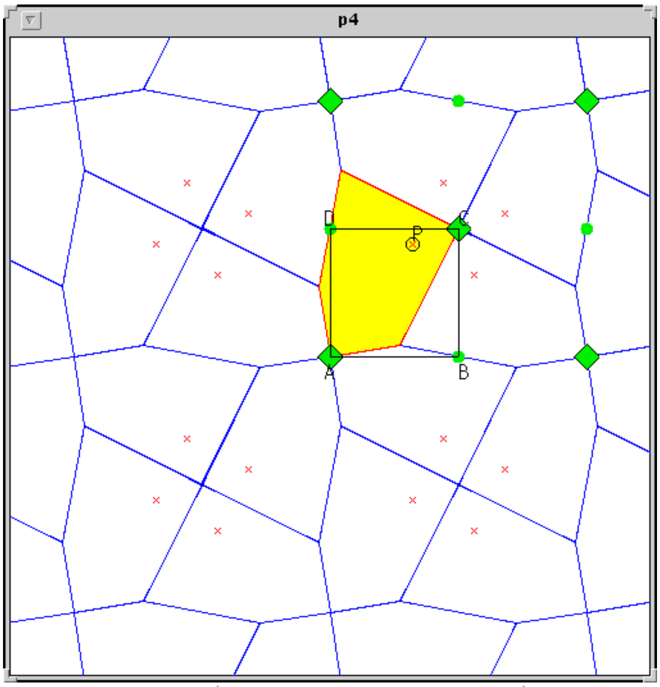

Fig. 3 arbitrary position (neither of the above cases) then the shaded pentagon is the Dirichlet domain.

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