Variational Surface Design and Surface Interrogation

H. Hagen
FB Informatik

Universität Kaiserslautern

Abstract

The generation of technical smooth surfaces from a mesh of three-dimensional data points is an important problem in geometric modelling. In this publication we give a survey of some new techniques based on a calculus of variation approach. Apart from the pure construction of these surfaces, the analysis of their quality is equally important in the design and manufacturing process. Generalized focal surfaces are presented here as a new surface interrogation tool.

0 Introduction

Curves and surfaces designed in a Computer Graphics environment have many applications, including the design of cars, air planes, shipbodies and modeling robots. The generation of "technical smooth" surfaces which are appropriate for the NC-process from a set of three-dimensional data points is a key problem in the field of Computer Aided GeometricDesign. The fundamental idea for the described methods is the use of modeling tools which minimize a certain functional that can be interpretated in the sense of physics and/or geometry. In chapter 1 we deal with a variational design method for B-Spline-Surfaces. This concept is extended to NURBS-surfaces in chapter 2.

In chapter 3 we present generalized focal surfaces as a new tool for surface interrogation.

1 Variational design of B-spline surfaces

In this section we leave the classical approach

- construct a smooth net of curves
- add the surface patches smoothly into the net

and present a direct method to construct a technically smooth B-Spline spline surface, which uses only point data and refrains from determining a net.

The construction algorithm combines a weighted least square approximation with automatic surface smoothing. The smoothing criterion is the approximate minimization of the curvature variation. This technique presented here aims at constructing tangent-plane continuous B-spline surfaces. The following mathematical model serves as variation principle:

$$
\begin{aligned}
&(1-ws) \quad \left\{\sum_{k=1}^{n_p} w_{pk}[X(u_k,v_k)-P_k]^2\right\} \\
+ \quad &ws \quad \left\{\sum_{i=1}^{n}\sum_{j=1}^{m} w_{3_u}\int_{v_j}^{v_{j+1}}\int_{u_i}^{u_{i+1}} w_{3_{u_{ij}}}\left\|\frac{\partial^3 X(u,v)}{\partial u^3}\right\|^2 dudv\right. \\
&\qquad \left. +w_{3_v}\int_{v_j}^{v_{j+1}}\int_{u_i}^{u_{i+1}} w_{3_{v_{ij}}}\left\|\frac{\partial^3 X(u,v)}{\partial v^3}\right\|^2 dudv\right\} \rightarrow min. \qquad (1)
\end{aligned}
$$

$X(u,v)$ is the representation of the surface, $(u,v) \in [u_1, u_{n+1}] \times [v_1, v_{m+1}]$ is the parameter value and n, m are the number of segments in u- and v-direction.

Applications We use this method to construct reflection surfaces for car headlights.

Step 1. Digitizing

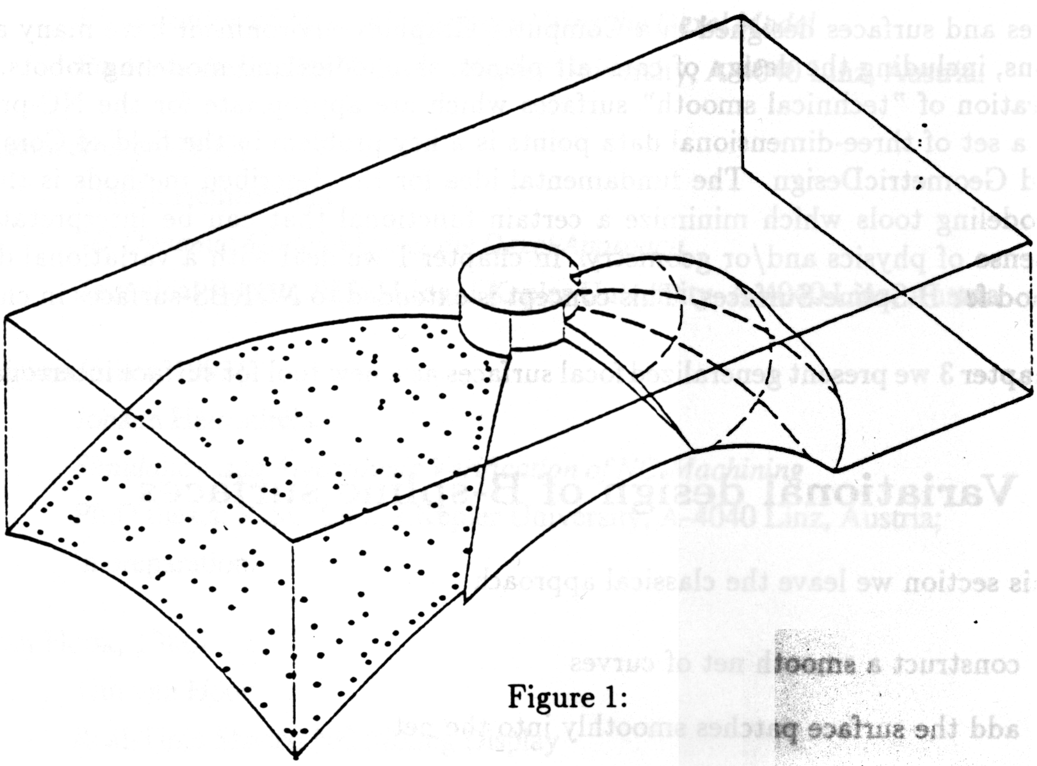

Figure 1:

Step 2. Parametrization

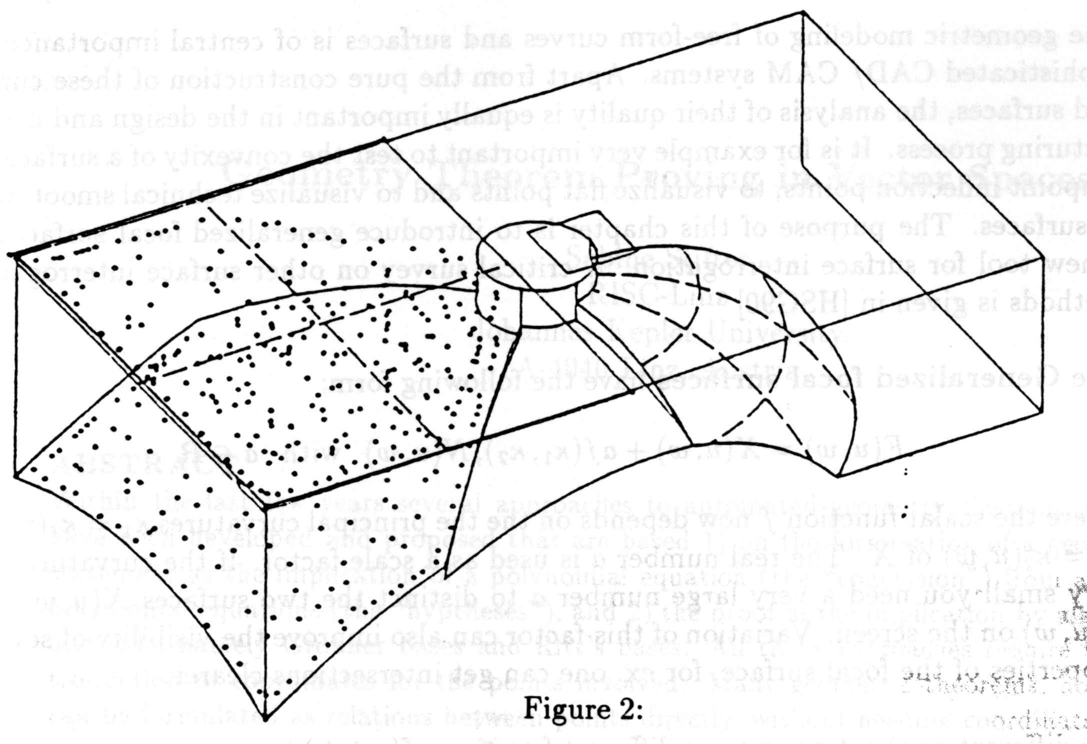

Figure 2:

Step 3. Variational Surface Design

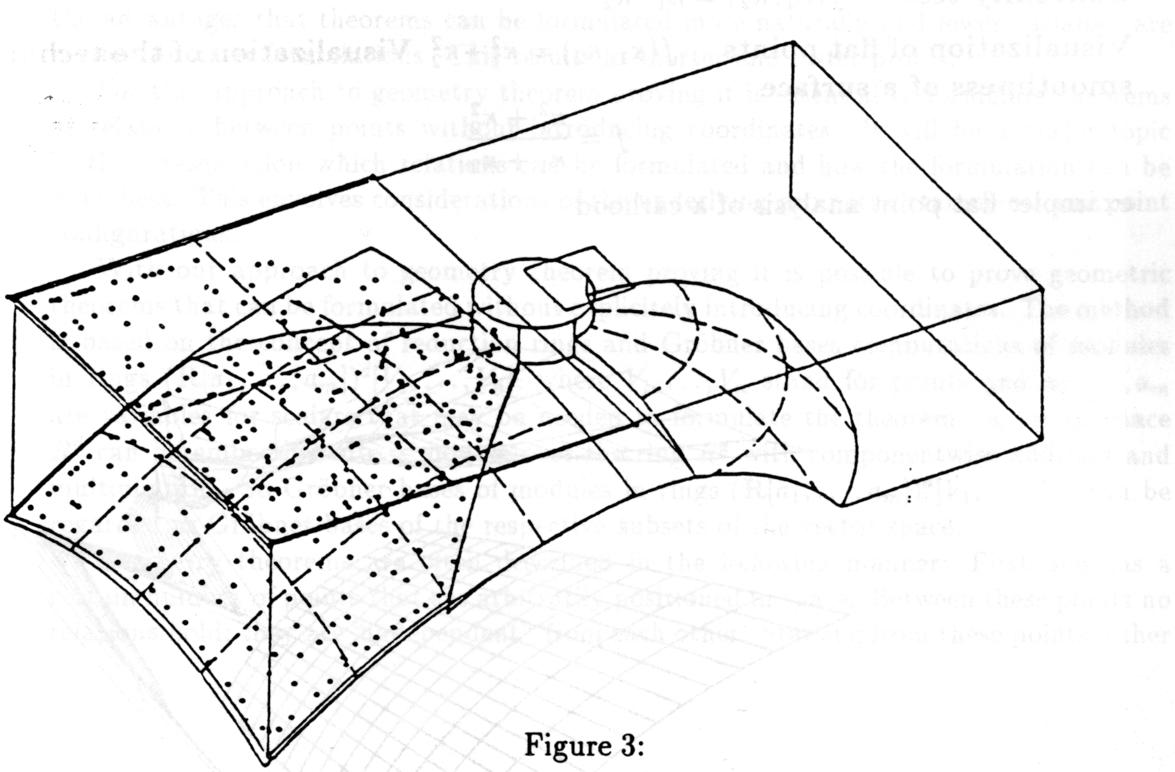

Figure 3:

This concept is extendet to NURBS in the next chapter.

2 Surface Interrogation

The geometric modeling of free-form curves and surfaces is of central importance for sophisticated CAD/ CAM systems. Apart from the pure construction of these curves and surfaces, the analysis of their quality is equally important in the design and manufacturing process. It is for example very important to test the convexity of a surface, to pinpoint inflection points, to visualize flat points and to visualize technical smoothness of surfaces. The purpose of this chapter is to introduce generalized focal surfaces as a new tool for surface interrogation. A critical survey on other surface interrogation methods is given in [HSG90]

The **Generalized focal surfaces** have the following form:

$$F(u,w) = X(u,w) + af(\kappa_1,\kappa_2), N(u,w) \quad \text{with} \quad a \in \mathbf{R} \tag{2}$$

where the scalar function f now depends on the the principal curvatures $\kappa_1 = \kappa_1(u,w)$, $\kappa_2 = \kappa_2(u,w)$ of X. The real number a is used as a scale factor. If the curvatures are very small you need a very large number a to distinct the two surfaces $X(u,w)$ and $F(u,w)$ on the screen. Variation of this factor can also improve the visibility of several properties of the focal surface, for ex. one can get intersections clearer.

For different applications we use different functions $f(\kappa_1,\kappa_2)$:

Convexity test $f(\kappa_1,\kappa_2) = \kappa_1 \cdot \kappa_2$

Visualization of flat points $f(\kappa_1,\kappa_2) = \kappa_1^2 + \kappa_2^2$ **Visualization of the technical smoothness of a surface**

$$f = \frac{\kappa_1^2 + \kappa_2^2}{\kappa_1 + \kappa_2}$$

example: flat point analysis of a carhood