Visualization of Algebraic Surfaces Using the Dexel Model

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Extended Abstract

The realistic and efficient visualization of algebraic surfaces together with basic operations (e. g. the intersection of two surfaces) is very helpful for the study of algebraic surfaces. An algebraic surface usually is displayed as a line drawing. For a realistic visualization it is desirable to generate a shaded image of the surface. Our idea was to represent the surface with a geometric modeler. In this way not only an efficient visualization can be done but also the advantages of the modeler like zooming, several windows with different view points, and different light sources can be used.

1 The Dexel Model

The dexel model has been developed in [van Hook, 1986] and further been investigated in [Heinzelreiter, 1990] and [Heinzelreiter, 1992]. The name dexel is - in keeping convention with the names pixel (for picture element) and voxel (for volume element) - the abbreviation for depth element.

This modeling scheme decomposes an object as follows: the object to be encoded is intersected by a set of parallel, equidistant rays. Two intersection points defining a line segment that lies fully inside the object make up a dexel. All dexels belonging to one ray are sorted and concatenated to a dexel list. All these dexel lists are organized in a dexel matrix. Figrue 1 shows the dexel encoding of an object.

Generally the dexel list is a list of intervals. In case of a surface the intervals have length zero, i.e. collapȅe to single points. For the visualization of the internal model only the endpoints are relevant. So no problems arise when an interval collapses to a point.

The boolean operations on dexel enoceded objects are reduced to the corresponding operations on sets of intervals. Because of the approximation that is done in the modeling step one has to care about the errors the endpoints of the intervals are afflicted with.

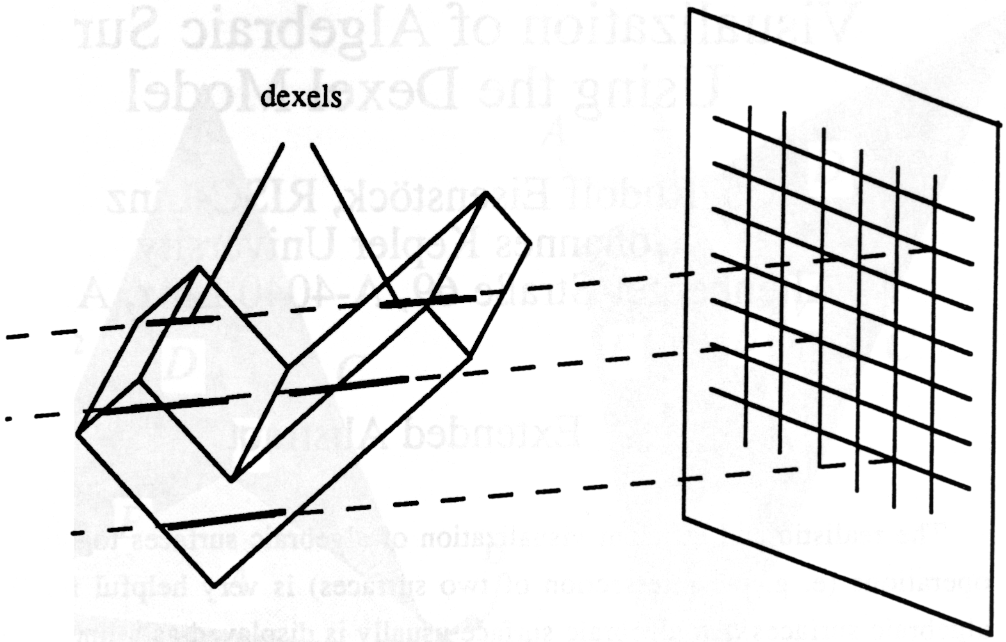


Figure 1: Dexel encoding of a simple object

2 Encoding a Surface into the Dexel Model

To encode an object into the dexel model means to intersect the object with a set of parallel straight lines. In case that the object is an algebraic surface we have two possibilities for description.

- Surface Given in Implicit Form:

In this case we specify the straight line L in explicit form:

x = f(t),

y = g(t),

z = h(t),

where f, g, h are polynomials linear in t.

Substituting L into S yields

F(f(t), g(t), h(t)) = 0.

This is a polynomial equation in one variable. Its degree is equal to the total degree of S. Solving the equation for t and substituting it into L gives the solutions for x, y, z.

In case that F(f(t), g(t), h(t)) is identically 0 (i. e. all (x, y, z) Œ R^3 are solutions) the dexel is the whole straight line L.

- Surface Given in Parameter Form:

In this case we specify the straight line L in implicit form, i. e. as the intersection of two planes:

U(x, y, z) = 0,

V(x, y, z) = 0,

where U, V are polynomials of degree 1.

Substituting S into L yields

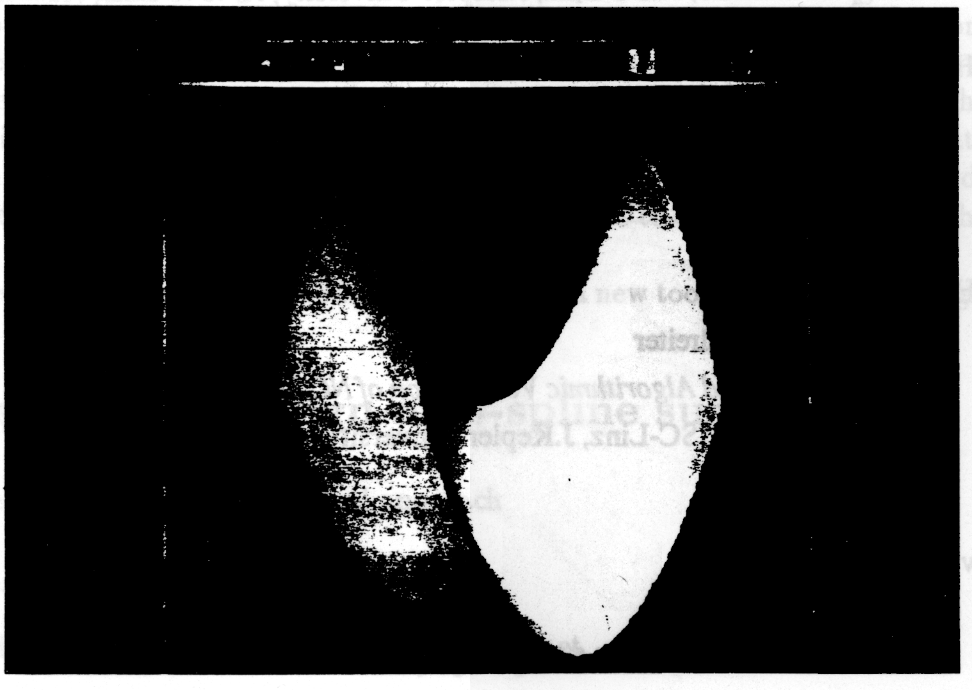
$$U\left(\frac{f_1(s,t)}{f_2(s,t)}, \frac{g_1(s,t)}{g_2(s,t)}, \frac{h_1(s,t)}{h_2(s,t)}\right) = 0,$$

$$V\left(\frac{f_1(s,t)}{f_2(s,t)}, \frac{g_1(s,t)}{g_2(s,t)}, \frac{h_1(s,t)}{h_2(s,t)}\right) = 0.$$

A possibility to compute the solutions of these two equations in s and t is to use Gröbner bases. In the case that the system has only finitely many solutions, the Gröbner basis contains one polynomial depending only on one of the variables. Otherwise either the whole line L is the dexel or L does not intersect S. (The two latter cases can also distinguished by the form of the Gröbner basis.)

To get a realistic visualization a grid of 200 x 200 lines is necessary. Therefore 40000 lines have to be intersected with the surface. So some effort has to be undertaken on an efficient intersection procedure. Since our first aim was to show the suitability of the model for visualizing algebraic surfaces, we restricted to surfaces of total degree 3.

The encoding of a surface is done w.r.t. a specific view point. By the view point a view plane is defined (the coordinate vector of the view point is the vector normal to the view plane). All rays are normal to the view plane. The following picture gives an example for a dexel encoded surface. The surface in implicit form is: $x^2 - y - z^2 = 0$.



3 Applications of the Dexel Model

- **Modeling of objects that may change their shape.**

For the visual simulation of NC-machining arise two requirements for the internal model: Fast performance of boolean operations and efficient visualization. These are two of the features of the dexel model.

- **Visualization of algebraic surfaces.**

As described above the dexel model is well suited for modeling and visualizing algebraic surfaces [Eisenstöck, 1992].

- **Image processing.**

Also for image processing-problems the dexel modell is suited. The probelm:

given: a set of points in three dimensional space, taken by two video cameras

find: a solid model of the scenarion defined by the points

is solvable by using dexels. The given points can be interpreted as endpoints of dexels.

4 Bibliography

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